Faraday's law—Quantitative experiments

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IV. SUMMARY

Using the complex numbers as a model, the perplex numbers $t + hx$, where $h^2 = 1$, define a unit curve consisting of two hyperbolas whose common asymptotes are interpreted as the light cone. By means of this unit curve the functions cosh, sinh, and tanh are extended from the set of real numbers R to the set $\Phi = R \times K$, where $K = \{1, h, -1, -h\}$. For $\theta \in R$, $k \in K$, the angle $\theta_k \in \Phi$ is then a velocity parameter such that $v = c \tanh \theta_k$. The fact that both θ_k and θ_{-k} give the same v then accounts for the different approaches in the literature to superluminal phenomena. The existence of two angles for the same v means, first, that a world line is to be thought of as being directed, and, second, that there are two superluminal transformations available, each of which corresponds to being in a superluminal reference frame with velocity v relative to the original frame. The difference in these two is that one generates a time reversal with regard to a given phenomenon while the other generates a space reversal. The different approaches in the literature tend to pick just one of these. It is suggested here that either both transformations should be considered or neither.

[1] Part of the fun in such a venture is naming things. Borman liked to call these numbers the "ineptitudes." Since I am writing this article I am going to use my favorite name and call them the "perplexes."

[2] At this point it is seen that "h" really stands for "hyperbolic." In fact there are two meanings of "hyperbolic" which seem appropriate. One, of course, has to do with hyperbolas, as in the unit curve. The other has to do with hyperbole, which, in literary parlance, means exaggeration, such as asserting something considered impossible to actually be the case. In this sense, the assertion $|h| = -1$ was hyperbole. Hyperbole of this sort is not uncommon in mathematics and is one method for generating new kinds of math systems.

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Faraday's law—Quantitative experiments

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A cylindrical bar magnet is dropped through a coil which encircles a glass tube. A Commodore 64 computer equipped with an analog to digital converter digitizes and stores the resulting voltage pulse. The magnet velocity and the number of turns on the coil are varied and simple measurements on the voltage pulse give quantitative tests of Faraday's law. Integration produces the total flux through the coil. Flux integral calculations which treat the magnet either as a dipole or as a uniformly magnetized cylinder lead to equations for the voltage pulses which give a good fit to the experimental data obtained with a set of variable area coils. Either model is good for radial distances > 40% the magnet length. Other results include determination of dipole separation and pole strength and surface current density. A versatile program for data acquisition and inspection is described. Suggestions are made for demonstrations and laboratory work for students in introductory noncalculus and calculus based courses and for advanced experimental work for majors.

I. INTRODUCTION

The main impediment to quantitative experiments with Faraday's law has been the lack of a suitable way to observe the induced emf. The galvanometer, the storage oscilloscope, and the ballistic galvanometer come to mind, but all suffer disadvantages—either of cost, speed, accuracy, or ease of use. A better instrument is a computer equipped with an analog to digital converter (ADC). This article reports our efforts to use the computer as a laboratory instrument whose purpose is to capture and store the voltage transients produced in experiments with Faraday's law and to make the computer transparent to the student so the essential phenomena under study are not obscured by instrumental or programming complications. Some alternative ways to study electromagnetic induction are reported.[1-3]

II. EXPERIMENTAL

A. Apparatus

A 1.0-cm-diam glass tube 125-cm long is supported vertically by a steel rod and meter stick as shown in Fig. 1. A lucite coil form with an axial hole for the glass tube was machined with grooves 4-mm wide and 1.59 cm in diameter on which were wound coils of 5, 10, 15, 20, and 25 turns using #36 AWG wire. An earlier version used 10, 20, 40, and 80 turns of #28 AWG wire. Coils wound with many turns of the thicker wire have larger diameters and make tests of voltage dependence on number of turns less satis-
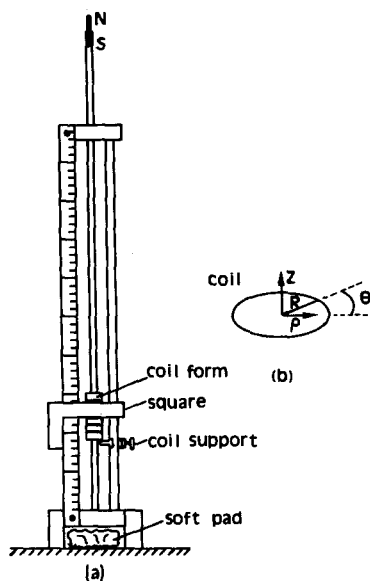
Fig. 1. (a) 1.0-cm-diam glass tube 125-cm long guides magnet fall. Lucite coil form has 5-, 10-, 15-, 20-, and 25-turn, 1.59-cm-diam coils of #36 AWG magnet wire. Coils may be adjusted vertically, position read with square placed across meter stick. (b) Center of magnet falls a distance z to coil.

factory. For variable coil area studies, a 20-turn coil of #28 AWG was wound on the glass. Similar coils with diameters 1.59, 2.24, and 3.19 cm were wound on another lucite coil form.

Silicone caulk kept the wire in place. A horizontal peg held by an apparatus clamp supports the coil form and lets it be adjusted easily along the tube. A small carpenter's square placed against the meter stick allows the coil position to be read.

A cylindrical magnet 6.7 mm in diameter and 4.5-cm long with a maximum B field of about 1000 G is placed with its center at the top of the tube and released. The magnet falls a distance z, Fig. 1, and is caught by hand or allowed to land on a piece of bubble wrap to avoid impacts which might change its magnetization.

The voltage pulse induced in the coil is fed to a Commodore 64 computer equipped with a linear amplifier and an ADC, as shown in Fig. 2. This computer interface is similar to those reported previously[4,5] and is available commercially.[6] An amplifier gain of five or ten is adequate for these experiments. The ADC samples the amplified pulse 29 000 times/s, converts the voltage sample to an 8-bit binary number, and the computer stores the number. With a gain of ten, this interface allows an input voltage range of − 150 to + 150 mV with a resolution of 1.17 mV (0.4%).
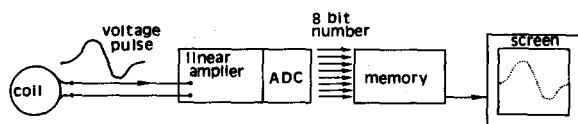

Fig. 2. Falling magnet induces voltage pulse in coil (typically 200-mV p-p). Pulse is amplified five or ten times and sent to analog to digital converter (ADC). ADC captures 29 000 voltage samples/s. Up to 8192 samples are stored in computer memory and then may be directed to video screen or otherwise manipulated.
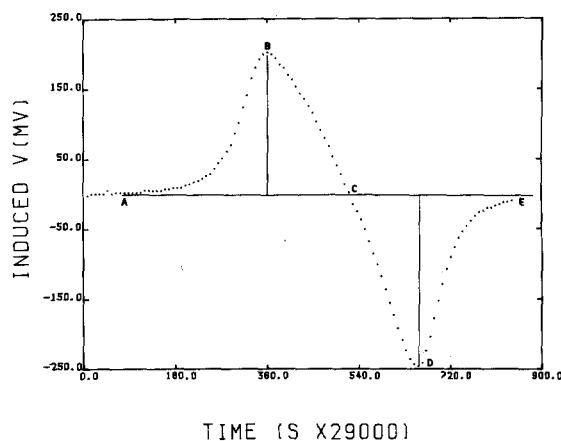
TIME (S X29000)

Fig. 3. Typical voltage pulse. AE is baseline. Features easiest to measure are voltages at peaks B and D and the time between peaks t_{BD}. ADC sample rate is 2.90×10^4 samples/s so the full scale time here is (900/ 29 000) = 31.0 ms. Only 120 of the 900 voltage samples are plotted.

B. A typical pulse

A typical voltage pulse is shown in Fig. 3 for a fall of 78 cm through a 40-turn coil. The horizontal axis is given in units of the ADC sample rate. Peak voltages are measured from the baseline AE. Approximately 900 samples were taken in about 30 ms. The second peak (D) is larger for two reasons: (i) because the magnet is going faster as the second pole passes through the coil and (ii) because of the magnet's field asymmetry (see Sec. IV).

The results of 20 drops of 55 cm through a 25-turn coil showed that the features of the pulse are reproducible to within about 0.8% in voltage and about 1.2% in peak-to-peak time.

C. Calibration

A known voltage, from a 1.5-V battery across a 1-K pot, is measured with a good digital multimeter and supplied to the ADC signal amplifier. The ADC reads the voltage and stores it with the DMM value which was typed in. When enough voltages have been read, say seven or eight, a linear least-square fit may be used with other programs to convert ADC readings into voltages. The amplifier and ADC circuits are linear to 0.4% and so measurements taken on a single gain setting would not need to be calibrated in terms of voltage if relative measurements would suffice.

Time calibration may be done by using the ADC to digitize a square waveform from a function generator. From the total number of voltage samples obtained for, say, 100 complete waveforms and the square wave frequency obtained from a frequency counter, the sample rate may be calculated. For this interface and its associated program the rate is 2.90×10^4 samples/s to within 0.3%.

III. THEORY

Faraday's law may be stated as follows[7]:

$$V = -N\left(\frac{d\Phi}{dt}\right), \tag{1}$$

where V is the induced voltage and N the number of turns in the coil. Φ is the magnetic flux through the coil, given by

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$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}, \tag{2}$$

where $\mathbf{B}$ is the magnetic flux density and $\mathbf{A}$ is the vector area of the circuit. If, for the geometry of Fig. 1 (b), a system of cylindrical polar coordinates ρ,θ,z is adopted, with z along the axis of the tube and magnet and normal to the coil, then Eq. (2) becomes

$$\Phi = \int_0^R B_z (2\pi\rho)d\rho, \tag{3}$$

where R is the coil radius. From the point of view of the coil, the falling magnet provides a time-dependent field which can be worked out with the chain rule:

$$\frac{dB_z}{dt} = \left(\frac{dB_z}{dz}\right)\left(\frac{dz}{dt}\right) \tag{4}$$

and dz/dt is just the magnet velocity v at the coil. So Eq. (1) becomes

$$V = -Nv \int_0^R \left(\frac{dB_z}{dz}\right)(2\pi\rho)d\rho. \tag{5}$$

In principle, if the details of B_z were known then the profile of the voltage pulse in Fig. 3 could be calculated. But even without knowing that, realizing that the integral in Eq. (5) is a *constant for any given position* of the magnet in the coil, we can make two predictions:

(1) The induced voltage is proportional to the number of turns N.

(2). The induced voltage is proportional to the magnet velocity v.

Of course, these are two of the propositions given in any textbook treatment of Faraday's law. A third proposition, that the induced voltage is proportional to the area of the coil, cannot be isolated and tested with this apparatus because the B field varies rapidly in the radial direction. Instead, a more comprehensive study is possible (see Sec. V).

IV. RESULTS

A. Voltage versus N

Figure 4 presents the voltage pulses obtained for coils of fixed radius and 5, 10, 15, 20, and 25 turns. The falling

Fig. 5. Peak voltage from Fig. 4 is proportional to the number of turns N. Lines are least-square fits. There are separate lines for the B and D peaks (Fig. 3) because this particular magnet has asymmetric poles.

distance z was constant at 109.5 cm resulting in a constant velocity, at the coil center, of

$$v = \sqrt{2gz} = 463 \text{ cm/s}, \tag{6}$$

where $g = 980 \text{ cm/s}^2$ is the acceleration due to gravity. Figure 5 plots induced voltage V versus N for the low velocity (B) peak and the higher velocity (D) peak. The lines are least-square fits and it is clear that V is proportional to N as predicted.

A cursory inspection of Fig. 5 might lead to acceptance of the notion that, since there are two voltage peaks, there need to be two lines, one for low velocity and the other for the higher velocity. But since the point of the experiment is to show that voltage is proportional to velocity, why do not *all* the points fall on one line?

A simple experiment showed why. The magnet used in this experiment was dropped twice, once with the north end up and once with the south end up. The result was that the north pole always produced the larger pulse. This magnet, as do several others tested with this apparatus, exhibits pole strength asymmetry.
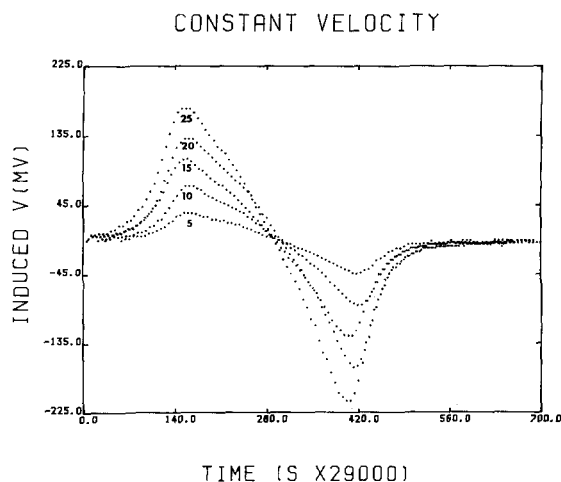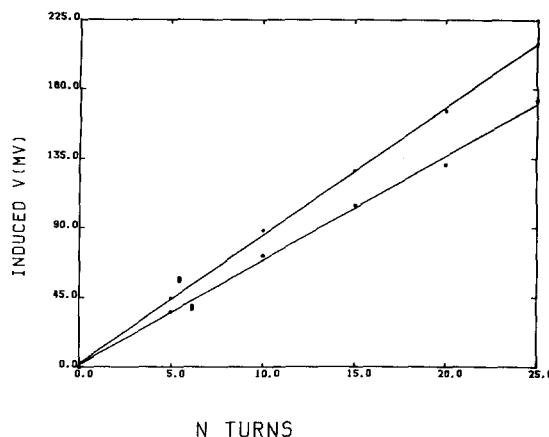
Fig. 4. Voltage pulses from coils of radius 0.79 cm and 5, 10, 15, 20, and 25 turns of #36 AWG wire. The magnet was traveling 463 cm/s after falling 109.6 cm.
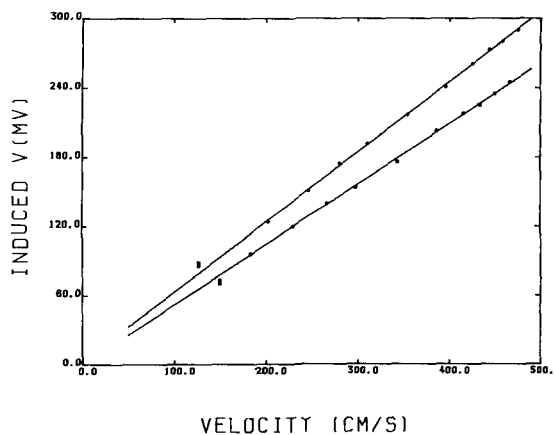
Fig. 6. Peak voltage is proportional to velocity for ten drops. Lines are least-square fits. Velocities have been adjusted according to Fig. 7 and Eqs. (7) and (8).
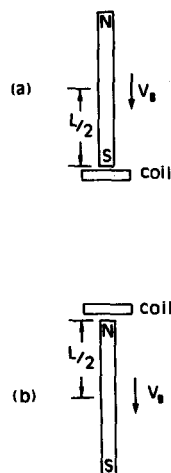
Fig. 7. If half the magnet length L is subtracted from (a), or added to (b), the dropping distance z, the better estimates of velocity from Eqs. (7) and (8) produce the least-square lines of Fig. 6.

B. Voltage versus velocity

Figure 6 plots induced voltage V versus velocity for ten values of z for the low velocity (B) peak and higher velocity (D) peak. Again, the lines are least-square fits and evidently V is proportional to v as predicted.

The velocities in Fig. 6 have been *adjusted*. If the raw velocities and voltages taken directly from the digitized voltage pulses are plotted, a straight line will fit most of the points fairly well. But as the velocity gets higher, the points depart more and more from a linear plot.

The adjustment is sketched in Fig. 7. In the first approximation, the distance of fall z used to calculate the velocity is measured from the coil to the center of the magnet in its initial position, Fig. 1. The south pole enters the magnet first, Fig. 7 (a), and causes the lower velocity (B) peak of the pulse while z is approximately half the magnet length L away from the coil. Hence, a better estimate of the velocity is given by

$$v_B = \sqrt{2g(z\text{-}L/2)} \ . \tag{7}$$

Similarly,

$$v_D = \sqrt{2g(z+L/2)} \ . \tag{8}$$

V. FURTHER ANALYSIS

A. Magnet effective length

The notion of adjusting the velocity for the position of the magnet in the coil, as given in Eqs. (7) and (8), may be extended as follows. Suppose that l is the actual distance between the poles, with $l < L$. Call l the "effective length." The poles cannot really be localized, of course, but we can speak as through they can to prepare for a test of the dipole approximation to the field (see Sec. V C).

With reference to Figs. 3 and 7, the time between the B and D peaks on the voltage pulse may be calculated as

$$t_{BD} = t_D - t_B = \sqrt{2/g}(\sqrt{z + l/2} - \sqrt{z - l/2}) \doteq l/\sqrt{2gz} \ . \tag{9}$$

Equation (9) predicts that a graph of the time between peaks t_{BD} vs $z^{-1/2}$ will yield a straight line with slope $l/\sqrt{2g}$. As Fig. 8 clearly shows, such a plot does give a straight line. The slope yields $l = 3.94$ cm for the effective length, i.e., the dipole separation.[8]
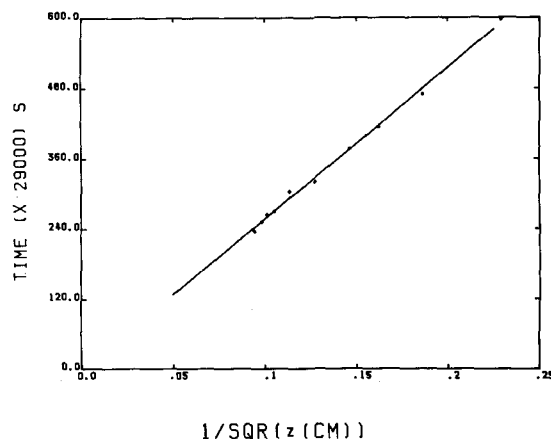
MAGNET EFFECTIVE L

Fig. 8. Time t_{BD} between peaks for data of Fig. 6 plotted against $z^{-1/2}$. Slope is $l/\sqrt{2g}$, where l is the dipole separation, Eq. (9).

B. Flux calculation

When the magnet is far from the coil the flux through the coil is negligible and so we should expect that integration of Eq. (1),

$$\Phi = \frac{1}{N} \int V \, dt \, , \tag{10}$$

would reflect this fact. It does, as Fig. 9 shows. This is not an obvious outcome when just based on inspection of Fig. 3, since the voltage pulse is asymmetric.

The maximum flux is about 1135 μWb. Ponder for a moment—what is the effect of magnet velocity on this flux profile, and on the maximum flux? The flux duration decreases as the velocity increases, just as you would expect, but integration shows that the peak flux is the same. This makes sense, of course, because when the magnet is in the coil there is flux through the coil whether there is motion or not.

C. Variable coil area experiments

The field from a bar magnet decreases rapidly with the radial distance and so experiments with coils of different areas will not show proportionality between the induced voltage and coil area. But it is possible to show that if the
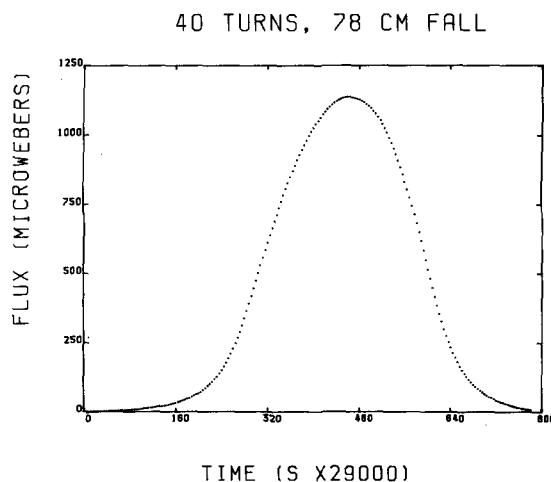

40 TURNS, 78 CM FALL

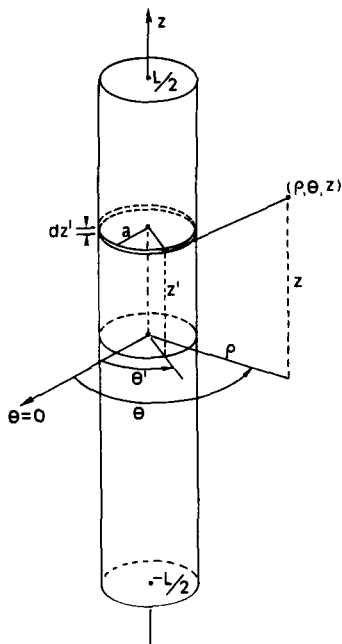Fig. 9. Pulse of Fig. 3 integrated to give flux.

Fig. 10. Coordinate system for calculations treating magnet as a uniformly magnetized cylinder equivalent to a cylindrical current sheet. Current element is a loop of thickness dz' a distance z' from magnet center. Magnet surface has (a,θ',z') axes. B field points use (ρ,θ,z) system.

ratio R/L of coil radius to magnet length is greater than about 0.4 then the field of a bar magnet approximates either (i) the field of a uniformly magnetized cylinder, or (ii) a dipole field. These two models require two methods of calculation.

1. Method one

The first method is illustrated by Elliott.[9] The magnet is assumed to have uniform magnetization M along its axis. The B field may be calculated from the curl of the vector potential, and results in a volume integral and a surface integral. The volume integral contains the curl of M and so is zero. The surface integral contains $M \times d S$ and this is zero on the ends. That leaves an integral over the cylindrical surface. This can be shown to be equivalent to a long thin solenoid with many close turns, or a current sheet. Elliott[9] and Smythe[10] work out the axial B field inside and outside the solenoid and leave it at that. Jackson[11] works out the off-axis field of a single current loop in a plane. He expresses the field in both elliptic integrals and in spherical

harmonics. Presumably the calculation could be generalized to a stack of current loops to form a cylinder and hence calculate the desired field. Our interest in this method dwindled, particularly since the whole thing depends on a *uniformly* magnetized cylinder, possibly a weak assumption with off the shelf bar magnets.

But then a paper by Dasgupta[12] appeared, in which he pointed out that the whole calculation could be done with the Biot–Savart law and multiple integrals, Fig. 10. He also pointed out two important simplifications: (i) that the field is axially symmetric and so the angle θ to the field point may be set to 0 without loss of generality, and (ii) that for the case of an infinitely long solenoid, the integration can be accomplished with a standard table of integrals provided the z integration is done before the θ' integration. His method may also be used to calculate the field anywhere from a finite solenoid and hence the field of the uniformly magnetized cylinder. However, the integral which results describing the voltage pulse does not seem to yield to elementary methods and so a numerical double integration is required. The calculation and results are described next.

Dasgupta assumed the solenoid to be a cylindrical current sheet of radius a with its axis along the z direction of a (ρ,θ,z) cylindrical coordinate system, Fig. 10. The magnetic material is described with the system (a,θ',z'). Assuming n turns per unit length, each carrying current I in the θ direction, then the surface current density is $K = nI$.

Dasgupta shows that the z component of the field dB_z due to a ring of current at $z = z'$ is given by

$$dB_z = \frac{\mu_0 K dz' a}{2\pi} \int_0^\pi \frac{(a - \rho \cos \theta')d\theta'}{[\alpha^2 + (z-z')^2]^{3/2}}, \tag{11}$$

where

$$\alpha^2 = \rho^2 + a^2 - 2\rho a \cos \theta'. \tag{12}$$

Integration over the length of the solenoid from $-L/2$ to $+L/2$ yields

$$B_z = \frac{\mu_0 K a}{2\pi} \int_0^\pi \frac{(a - \rho \cos \theta')}{\alpha^2}$$
$$\times \left(\frac{(L/2 - z)}{\sqrt{\alpha^2 + (z - L/2)^2}} + \frac{(L/2 + z)}{\sqrt{\alpha^2 + (z + L/2)^2}} \right) d\theta'. \tag{13}$$

Differentiation with respect to z and substitution into Eq. (5) results in

$$V(z,\theta = 0) = Nv\mu_0 K a\left(\int_0^R \int_0^\pi \frac{(a - \rho \cos \theta')}{\alpha^2} \left\{ \left[\alpha^2 + \left(z + \frac{L}{2}\right)^2\right]^{-1/2} \right. \right.$$
$$- \left[\alpha^2 + \left(z - \frac{L}{2}\right)^2\right]^{-1/2} - \left(z + \frac{L}{2}\right)^2 \left[\alpha^2 + \left(z + \frac{L}{2}\right)^2\right]^{-3/2}$$
$$\left. \left. + \left(z - \frac{L}{2}\right)^2 \left[\alpha^2 + \left(z - \frac{L}{2}\right)^2\right]^{-3/2} \right\} d\theta' \rho \, d\rho \right) \tag{14}$$

as the equation for the voltage pulse.

This was numerically integrated with a program in BASIC. Each coil required a 70-min computation to evaluate the voltage at 30 values of z. The dipole calculation given next takes only 3 min.

2. Method two

This method assumes poles of strength p and separation l and the inverse square law. This was popular in older texts, for example, Sears and Zemansky.[13] Cullity[14] gives the MKS version of the B field from a pole as
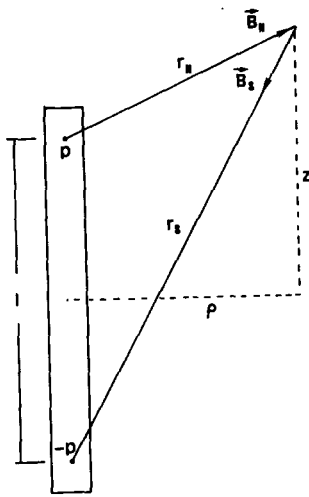
Fig. 11. Coordinate system for dipole calculation, Eq. (16). Poles of strength p are separated a distance l.

$$B = P/4\pi d^2 \tag{15}$$

and so the vector magnetic induction **B** for the dipole, Fig. 11, is

$$\mathbf{B} = (p/4\pi)(\mathbf{e}_N/r_N^2 + \mathbf{e}_S/r_S^2) , \tag{16}$$

where $\mathbf{e}_N$ is a unit vector pointing away from the north pole and $\mathbf{e}_S$ points toward the south pole. This yields

$$B_z = \left(\frac{p}{4\pi}\right)\left[\left(z - \frac{l}{2}\right)\Big/ r_N^3 - \left(z + \frac{l}{2}\right)\Big/ r_S^3\right] . \tag{17}$$

Substitute this into Eq. (3), integrate over ρ from 0 to the coil radius R, and differentiate with respect to z to evaluate Eq. (5). The result is

$$V = \left(\frac{Nvp}{2}\right)\left\{\left[R^2 + \left(z - \frac{l}{2}\right)^2\right]^{-1/2}\right.$$
$$- \left(z - \frac{l}{2}\right)^2\left[R^2 + \left(z - \frac{l}{2}\right)^2\right]^{-3/2}$$
$$- \left[R^2 + \left(z + \frac{l}{2}\right)^2\right]^{-1/2}$$
$$+ \left.\left(z + \frac{l}{2}\right)^2\left[R^2 + \left(z + \frac{l}{2}\right)^2\right]^{-3/2}\right\} . \tag{18}$$

If the angular variation in Eq. (14) is ignored, the z dependence is the same as Eq. (18).

3. Results

Voltage pulses were digitized for two magnets as each fell 115.6 cm through four coils with $N = 20$ turns and radii 0.5, 0.79, 1.12, and 1.59 cm. One magnet was 4.5-cm long and had a radius of 0.335 cm. The other was 2.0-cm long and had a trapezoidal cross section, approximated as a cylinder of radius 0.2 cm. Voltage pulse shapes were calculated using both the dipole model and the uniformly magnetized rod model. The actual magnet lengths, 4.5 and 2.0 cm, as well as the effective lengths, 3.94 and 1.8 cm (see Sec. IV), were used.

An example of results is given in Fig. 12. Calculations for either model gave indistinguishable shapes, except at the two peaks. The magnetized rod model gives results near
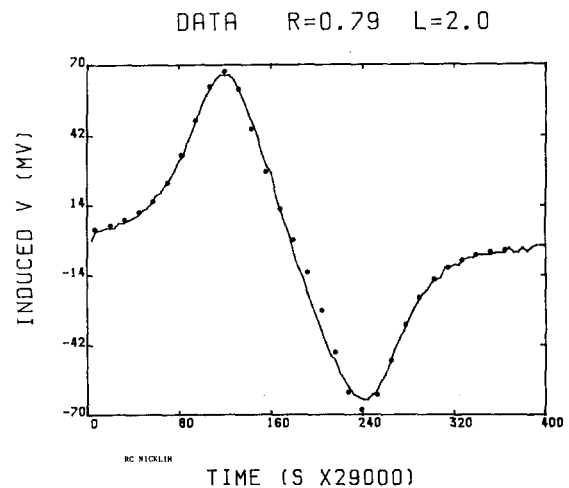

Fig. 12. Solid line is pulse from a 2.0-cm-long magnet falling through a 0.79-cm radius coil. Points were calculated with the dipole model, Eq. (18). Calculations with uniformly magnetized rod model give almost identical results except at the peaks, where z in Eq. (15) is close to $\pm L/2$. The close agreement between experiment and calculation verifies the flux integral part of Faraday's law, Eq. (2), and shows that either magnet model is valid to radii as small as 0.4L.

$z = \pm L/2$ which fluctuate rapidly because of the denominators, unless very small radial steps are taken.

A rule of thumb emerged: If $R/L > 0.40$, then either model will give a good fit to the voltage pulses, as shown in Fig. 12. This is surprising, because the usual assumption is that we need $R/L > 5$ or 10 to have useful field approximations from either model. If $R/L < 0.40$, then the fit with either calculation gets progressively worse.

That the voltage pulses can be synthesized by either model not only gives confidence in the models, but also means that Eq. (5), Faraday's law, has been experimentally tested in detail. Since either the pole strength or the current density factor in the models is simply a multiplicative constant, either can be determined from a fit of the experimental pulse for a magnet. However, determination of actual values for these constants is of little use unless, perhaps, one wants to compare different magnets. Certainly, the dipole model is easier to calculate and should be retained despite the theoretical importance of considering all magnetism as arising from current loops.

VI. SOFTWARE

One general purpose BASIC program, C64 FAST ADC, supports all the data acquisition and storage, and functions equally well for classroom demonstrations and student use in the laboratory. A menu allows single keystroke selection of instructions, change parameters, take data, display data (on 320×200 graphics screen), glass stripchart (to scan all the data), make disk file, and printout data. Send a blank disk if you would like a copy of C64 FAST ADC.

VII. PEDAGOGICAL SUGGESTIONS

There are three areas where Faraday's law discussions can benefit from an ADC equipped computer. These are demonstrations, introductory general physics labs, and advanced labs for physics majors.

A. Demonstrations

The computer can help you to show quickly: polarity effects, depending on which pole reaches the coil first; voltage versus N effects, Fig. 4; voltage versus velocity effects; effect of different magnets; the effect of putting two or more similar magnets together, end to end, and dropping them through the coil; and the short time (30 ms or so) it takes the magnet to fall through the coil.

B. Undergraduate laboratories

Students in a noncalculus course can do ten drops in 30 min, scan each data set with the glass stripchart and record their numbers, and move on to make some graphs to check out how the induced voltage depends on the number of turns, and on the velocity. These students probably have to be reminded how to calculate the velocity.

Students in the calculus based course can do all of the above and in addition may be encouraged to look for the $L / 2$ adjustment to the calculation of velocity. They might also use a canned least-square program running on another computer. Their voltage versus time data for a pulse could be printed out and they could do an integration by any of several fast methods to see that the flux comes back to zero. Another computer could be used to do this with a canned program, or to demonstrate either of the integrations over the coil area described in Sec. V. This might be the first practical application they will see of some of the concepts they labor over in their calculus classes.

If you have several computers, each can be equipped with an ADC. The falling magnet apparatus can be replicated inexpensively. If you only have one computer then you can do what you do for the laboratory with any other one-of-a-kind apparatus—schedule groups of students to use it, and have other experiments or activities scheduled to keep all your groups gainfully occupied.

C. Advanced undergraduate laboratories

Students can check out V versus N; V versus v; V versus coil area and do the dipole calculation of Sec. V; determine the dipole separation; write their own least-square program and use it; devise a launch method to produce constant, but known, velocities; wind their own coils, perhaps solenoids; go for bigger velocities—this apparatus should detect pulses at velocities at least ten times those used here; and test the effect of magnet length and radius on the fit to the

voltage pulse—this procedure can be streamlined to save them time.

VIII. SUMMARY

A personal computer equipped with an analog to digital converter and an applications program makes a very versatile laboratory instrument capable of digitizing voltage transients. In particular, a Commodore 64 was used to develop an experiment in which students may do straightforward tests of several aspects of Faraday's law of induction. The computer digitizes and stores the voltage pulse caused when a small cylindrical permanent magnet is dropped through a coil. Two common models of permanent magnets are shown to give the observed pulse shape.

Instructional uses include classroom demonstrations, laboratory work for students in both calculus and noncalculus based introductory physics courses, and open-ended experiments for advanced undergraduates.

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An experiment with two air tracks

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The analysis of a mechanical system with two air tracks is presented. The theoretical results are compared with the experimental data.

In this article we present a two-air track system experiment. To our knowledge, this particular experiment has not been discussed in the literature. A pedagogical merit of this experiment is that the theoretical analysis can be done on the level of general physics or undergraduate mechan-

ics. But the system is complicated enough to allow three different modes of motion. The theoretical position–velocity plot can be obtained numerically on a computer. The comparison of the experimental fits with the theoretical results shows a good overall agreement.